GCSE Maths – Algebra

nth term

Notes

WORKSHEET

n^{th} Term

The **formula** for the nth term of a sequence makes it **quick** and easy to find **any term** in the sequence.

Finding the 100^{th} or the 500^{th} term would take a long time by calculating each sequence value in turn, but using the nth term is quick and simple.

Using the nth term

The nth term formula contains the **parameter** $\boldsymbol{n}$ which denotes the **term number** in the sequence. For example, to find the **first term** in the sequence you would substitute $\boldsymbol{n = 1}$ into the formula. Similarly, for the **second term** you substitute $\boldsymbol{n = 2}$ into the formula, and so on. If you wanted to find the **85th term** in the sequence you would substitute $\boldsymbol{n = 85}$ into the formula.

Example: What is the 50th term of the sequence

$$2, 6, 10, 14, 18, \ldots$$

if the nth term is $\boldsymbol{4n - 2}$?

By inspection, the common difference is $+4$.

We can find the 6^{th} *term is* $18 + 4 = 22$.

Finding the 50^{th} *term in this way would take a long time so we instead use the formula for the nth term:*

Substituting $n = 50$ *into* $4n - 2$:

50th term: $4(50) - 2 = \mathbf{198}$.

We'll learn how to find the nth term later.

Example: What is the next term of the sequence 4, 7, 10, 13, 16, … with nth term $\boldsymbol{3n + 1}$? What is the 100th term of the sequence?

1. *By inspection, the common difference is* $+3$.
   The next term is $16 + 3 = \mathbf{19}$.

   We could have used the nth term formula to find the 6th term:

   Substitute $n = 6$ *into the nth term formula* $3n + 1$: $3(6) + 1 = \mathbf{19}$.

2. *For the 100th term it would take too long to keep adding 3 again and again. Instead use the nth term:*

   Substitute $n = 100$ *into the formula* $3n + 1$: $3(100) + 1 = \mathbf{301}$.

Finding the nth Term of Linear Sequences

Step 1: Find the **common difference** between terms and use it to make an **initial guess** for the nth term formula.

- If the common difference is 3, start with 3n.
- If the common difference is -4, start with -4n, and so on.

Step 2: **Compare** the initial guess of the nth term to the given sequence and **adjust** the nth term guess by adding a value which makes the two sequences match.

Example: Find the nth term of the sequence 5, 7, 9, 11, 13, …

1. Find the common difference and use this to make an initial guess for the nth term.

   The common difference is $7 - 5 = \mathbf{2}$*.*
   So, we start off with the nth term $\mathbf{2n}$*.*

2. Compare the initial guess of the nth term to the given sequence and adjust the nth term guess by adding a value which makes the two sequences match.

N:	1	2	3	4	5
2N:	2	4	6	8	10
	+3	+3	+3	+3	+3
sequence:	5	7	9	11	13

   Comparing the sequence with nth term $2n$ *to the given sequence, we must* $+3$ *to each term to obtain the given sequence.*

   Since each term of $\mathbf{2n}$ *needs* $+3$ *to make it into the given sequence, the nth term is*

$$\mathbf{2n + 3.}$$

Example: Find the 100th term of the sequence 6, 11, 16, 21, 26, ...

To find the 100th term, we must first find the nth term:

1. Find the common difference and use this to make an initial guess for the nth term.

   The common difference is $11 - 6 = \mathbf{5}$*.*
   So, we start off with the nth term $5\mathbf{n}$*.*

2. Compare the initial guess of the nth term to the given sequence.

5n	5	10	15	20	25
Sequence	6	11	16	21	26

   Each term of $5n$ *needs* $+1$ *to make it into the given sequence, so the nth term is* $\mathbf{5n + 1}$*.*

3. Find the 100th term.

   Substitute $n = 100$ *into the nth term:* $5(100) + 1 = \mathbf{501}$.

Finding the nth Term of Quadratic Sequences (Higher Only)

Non-linear sequences are sequences where the difference between terms is **not constant**. This means there is not common difference. **Quadratic sequences** are examples of non-linear sequences.

The **nth term** of a **quadratic sequence** has the general form $\boldsymbol{an^2+bn+c}$.

'a', 'b', and 'c' are constant and 'a' is generally not 0. If $a=0$ then the sequence is an arithmetic (linear) sequence as we saw on the previous page.

If you find the **sequence of differences** between terms of a quadratic sequence, the sequence of differences changes by the same amount each time.

Example: Find the nth term of the following quadratic sequence 3, 9, 19, 33, 51, ...

1. Work out the differences between the terms. Write the differences so that they form a new linear sequence.

   3, 9, 19, 33, 51, ...
   +6 +10 +14 +18

   Sequence of differences: 6, 10, 14, 18, ...

2. Use the term-by-term rule of the sequence of differences to find the coefficient of n^2.

   In the sequence of differences 6, 10, 14, 18, ..., the term-by-term rule is $+4$.

   Since the original sequence is a quadratic sequence, it will have an n^2 term in the formula. The coefficient of n^2 is always half of the term-by-term rule of the sequence of differences. In this case, the term-by-term rule is $+4$ so the coefficient of n^2 will be 2.

   Coefficient of n^2: $\mathbf{2}$

3. Compare the given sequence with the quadratic sequence __n^2 using the coefficient of n^2 found in the previous step.

$2n^2$	2	8	18	32	50
Sequence	**3**	**9**	**19**	**33**	**51**
Difference	+1	+1	+1	+1	+1

4. Find the linear part of the quadratic nth term by finding the linear nth term of the new sequence of differences.

   The new sequence of differences is 1, 1, 1, 1, 1, ...
   So, the linear nth term for the sequence of differences is simply $+1$ as each term in the sequence is the same.

5. Find the nth term of the quadratic sequence by combining the linear nth term of the sequence of differences found in step 4 and the coefficient of n^2 found in step 2.

   The linear nth term for the sequence of differences was $+1$ and the coefficient of n^2 was found to be 2. So, the nth term for the quadratic sequence is

$$\mathbf{2n^2+1}.$$

Example: Find the nth term of the following quadratic sequence -17, -30, -49, -74, -105 ….

1. Work out the differences between the terms. Write the differences so that they form a new linear sequence.

-17, -30, -49, -74, -105, ...

-13 -19 -25 -31

Sequence of differences: -13, -19, -25, -31, ...

2. Use the term-by-term rule of the sequence of differences to find the coefficient of n^2.

In the sequence of differences, the term-by-term rule is -6.

Since it is a quadratic sequence, it will have an n^2 *term in the formula. The coefficient of* n^2 *is always half of the term-by-term rule of the sequence of differences. In this case, the term-by-term rule is* -6 *so the coefficient of* n^2 *will be* -3.

Coefficient of n^2: -3

3. Compare the given sequence with the quadratic sequence __n^2 using the coefficient of n^2 found in step 2.

-3n²	-3	-12	-27	-48	-75
Sequence	***-17***	***-30***	***-49***	***-74***	***-105***
Difference	*-14*	*-18*	*-22*	*-26*	*-30*

4. Find the linear part of the quadratic nth term by finding the linear nth term of the new sequence of differences.

For the new sequence of differences -14, -18, -22, -26, -30, … the term-by-term rule is -4. *Comparing the sequence of differences with the sequence generated by the nth term* $-4n$, *there is a difference of* -10 *for each term. So, the nth term for the sequence of differences is* $-4n - 10$.

n	***1***	***2***	***3***	***4***	***5***
-4n	*-4*	*-8*	*-12*	*-16*	*-20*
-4n-10	***-14***	***-18***	***-22***	***-26***	***-30***

5. Find the nth term of the quadratic sequence by combining the linear nth term of the sequence of differences found in step 4 and the coefficient of n^2 found in step 2.

The linear nth term for the sequence of differences was $-4n - 10$ *and the coefficient of* n^2 *was found to be* -3. *So, the nth term for the quadratic sequence is*

$$\mathbf{-3n^2 - 4n - 10}.$$

n^{th} Term – Practice Questions

1. Find the nth term of the following sequences:

   a) 0, 4, 8, 12, 16, …

   b) -5, -3, -1, 1, 3, …

   c) -1, -5, -9, -13, -17, …

   d) 0, -13, -26, -39, -52, ...

2. For each of the following nth terms, give the first 5 terms and the 90th term of the sequence:

   a) $-6n + 4$
   b) $42n$
   c) $13n - 1.5$
   d) $-9n - 2$

3. Is 925 a term in the sequence -5, -1, 3, 7, 11, … ?

4. **(Higher Only)** Find the nth term of the following quadratic sequences

   a) -10, -21, -40, -67, -102, ….

   b) -2, 12, 34, 64, 102, …

5. **(Higher Only)** A sequence has an nth term of $6n^2 - 64n - 150$. Work out which term in the sequence has a value of -128.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.